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Nonlinear, Hyperbolic and Inverse Problems

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The final technical report consists of the annual reports for the period of the grant, 1984-1987, a report for the years 1988-1990 and this summary.

The principal investigator was Cathleen S. Morawetz and the other senior investigator was Louis Nirenberg 1984-1987. A number of research assistants and associates were also supported at various times, see annual reports.

Work was concentrated in the following areas.

C.S. Morawetz - Transonic flow, Direct and Inverse problems of wave propagation

L. Nirenberg - Nonlinear p.d.e. using topological methods

During the period of the grant C.S. Morawetz was elected to the National Academy of Sciences.

Report for 1988-90.

C.S. Morawetz was the only senior investigator.

A. Acoustic scattering

A study was made of the scattering by a smooth disturbance of the sound speed. The geometry was a half-plane. The signal was a blast on the surface of the plane and the changes in the sound speed ranged from cutting it to less than half to increasing it to double or more. The basic approach was to determine:

(1) The new ray surface.

(2) Use methods analogous to those in [1] to compute the disturbance off the ray surface at least as far as any caustic formation.

(3) Attempt to correlate scattered data on the boundary plane with the perturbation in the sound speed.

Students supported by AFOSR #549620-87-C-0065 performed the computations over three successive summers.

The computation of the ray surface and associated amplitude (to two terms in a singular expansion) involved solving 35 o.d.e. and could be done very accurately even up to the initiation of a single caustic.

The disturbance was computed. Because of the smoothness of the disturbance in the sound speed (5 continuous derivatives) the disturbance was very small but showed a quite distinct pattern. It was also moderately independent of mesh size.

The attempt to correlate was limited to showing that an L^2 computation of the disturbance was linear in a disturbance parameter. There was also linear correlation when the disturbance depended on two parameters.

This work has raised some interesting questions about whether the ill-posed inverse problem that goes with the forward problem could be attacked by some quite different methods since the reflection data up to a given time depends only on a segment of the perturbation in the sound speed. Reference [10]. This work was surveyed at Texas A&M conference, (March 1991) and Wave Propagation conference organized by INRIA at Strasbourg (April 1991).

B. Transonic flow

The proofs of the existence of a transonic flow past a bump in a box with added viscosity required a new model and the introduction of a barrier function to solve the partial differential equations and obtain flows with bounded velocity. This work is being written up as two papers, one on the viscous model which has some new approaches for a third-order system. And one on establishing the limit as viscosity tends to zero using new entropy pairs in a compensated compactness argument. An extension of Serre's use of triplets of entropy pairs was written up separately [8] and presented at Irsee conference, summer 1990.

The transonic flow existence was presented at the Birmingham P.D.E. conference (Feb. 1990) and will be presented at the Petrovsky conference in Moscow (June 1991).

C. Reflection of a plane shock by a wedge.

This problem can be studied by similarity solutions. A model for weak shocks was constructed making use of the fact that the reflected shock zero order in the strength of the shock is the transonic curve of the flow behind the shock. The reflections that occur are either weak or pseudo-Mach. The weak case studied by Keller, Blank, Hunter and Harebetian is reanalyzed using this idea. The case when weak reflection cannot occur cannot be replaced by a theory of

Mach reflection since this cannot take place in the weak shock range. Instead a theory is developed in which the impinging shock bends into a kind of Mach stem and behind it there is a compression wave (not completely analyzed) generated by a Burger-type equation. The loop is closed by the transonic shock described previously. This qualitatively checks with the computations of Glass and Colella.

This work has been described at a conference in Jerusalem, June 1990 and reported on.

D. Mixed equations in plasmas.

A new kind of singular phenomena for a Tricomi-like equation occurs in plasmas. The basic question is whether ill-posedness does not generate a singularity at a particular special point where the equation changes type in a particular way. A model problem was solved with Weitzner and Stevens [9].

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In Preparation:

1. R.V. Kohn and C.S. Morawetz, I. On existence of viscous solutions for transonic flow. II. Limit of viscous solutions.

2. Morawetz, C.S., On reflection of a weak shock by a wedge.